Patterns of Meaning of Students' Mathematical Experiences At University

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Abstract

This paper reports on the latest results of our research into the conceptions of mathematics, orientations to studying it and experiences of learning it of first year mathematics students at Sydney University. Questionnaires were issued to students at the beginning of the academic year and after one semester. An analysis of the results suggests two qualitatively different patterns of students' experiences of learning mathematics. Differences in students' conceptions and approaches were related to their examination performances.

Introduction

This research builds on our earlier phenomenographic study (Crawford, Gordon, Nicholas & Prosser, 1993, 1994a) which indicated that students enter university with a range of qualitatively different conceptions of mathematics and approaches to learning it. We found that over 75% of students conceive of mathematics as a fragmented body of knowledge, and learn it using repetitive and surface approaches. Further, our results showed that the ways in which students conceive of mathematics is related to their approach to learning. In discussing the results of that study we postulated that if the students perceive that the workload is too great, or believe that the assessment measures reproduction, then they are also likely to adopt low level conceptions and surface approaches to learning.

Phenomenographic research (Marton, 1988) describes the qualitatively different ways students relate to and understand phenomena. This approach views phenomena systemically and avoids the boundaries between person and context. This is consistent with a Vygotskian (1978) view that there is no assumption of a duality between self and context; between thinking and acting.

In this paper we extend our previous research using questionnaires developed from the earlier findings. We explore the relationships between how students conceive of mathematics on the one hand, and their approaches to learning mathematics and how they perceive their mathematical studies on the other.

The Questionnaires

Development of the Conceptions of Mathematics Questionnaire

In our earlier study, the conceptions of mathematics held by first year university mathematics students were identified from our analysis of students' written statements. These were open-ended responses to the question: *Think about the maths you've done so far. What do you think mathematics is?* The conceptions of mathematics are summarised in Table 1.

Table 1: *Conceptions of Mathematics*

Fragmented Conceptions
A. Maths as number, rules and formulae
B. Maths as numbers etc. with applications to problems
Cohesive Conceptions
C. Maths as a way of thinking
D. Maths as a way of thinking for complex problem solving
E. Maths provides new insights for understanding the world

The conceptions are described in detail in Crawford et al (1994a, p. 335). Here it is important to note the structural distinction between conceptions A and B on the one hand and conceptions C, D and E on the other. Conceptions A and B present mathematics as a fragmented body of knowledge, while conceptions C, D and E describe a cohesive view of mathematics. It should also be noted that the conceptions are logically and empirically inclusive, forming a hierarchy with conception A at the bottom of the hierarchy and conception E at the top.

These conceptions were used as the basis of the development of the Conceptions of Mathematics Questionnaire (CMQ). The written statements made by the students to the question given above were classified in terms of the conceptions. These statements were then used as the basis for developing a set of questionnaire items representing two broad conceptions — a fragmented and a cohesive conception.

The face validity of the items was examined by each author rating each item according to whether it suggested a fragmented or a cohesive conception. After comparing the ratings for each item among the authors, the items were revised. This resulted in the first trial version of the questionnaire. This version was piloted on a group of upper secondary mathematics students. After a series of internal consistency reliability and factor analyses, the questionnaire was further revised.

The final questionnaire is comprised of two subscales; a Fragmented Conception subscale and a Cohesive Conception subscale. Further details of the development of the questionnaire and the questionnaire itself are published in Crawford, Gordon, Nicholas and Prosser (1995).

The Modifications of the Study Process Questionnaire and Course Experience Questionnaire

Further information about students' approaches to studying mathematics was elicited using a modified version of the Study Process Questionnaire (Biggs, 1987). The modified questionnaire, the Mathematics Study Questionnaire (MSQ) is comprised of four subscales, measuring a surface intention, a surface strategy, a deep intention and a deep strategy when learning mathematics.

The students' accounts of their experiences of studying mathematics at university were elicited using a modified version of the Course Experience Questionnaire (Ramsden, 1990). This questionnaire was modified to fit the context of the students' mathematics subject. This modified questionnaire, the Experience of Mathematics Questionnaire (EMQ), is comprised of five subscales, measuring students' perceptions of the teaching and learning environment. These are their perceptions of: whether the workload is too high (Inappropriate Workload), whether the assessment is measuring reproduction (Inappropriate Assessment), how good the teaching is (Good Teaching), how clear the goals of the subject are (Clear Goals), and whether they have freedom in learning (Independence).

Data Collection

Data were collected from students enrolled in the first year course Mathematics 1 on five occasions. A detailed description of the data collections and the final data set on 300 students is given below.

1. The Conceptions of Mathematics Questionnaire (CMQ) and the Mathematics Study Questionnaire (MSQ) were administered to all students who attended the first mathematics tutorial in the first week of semester 1. The students were given 30 minutes of class time to complete these questionnaires, resulting in 1109 returns out of a total enrolment of approximately 1400.

2. A question asking students to explain their understanding of a mathematical concept was given as a part of the first assignment to all students with a surname beginning A-K. Approximately 500 students completed this assignment. Results from this aspect of the study have been reported in Crawford, Gordon, Nicholas and Prosser (1994b).

3. A slightly modified version of the same question was included in the Semester 1 examination.

4. The Conceptions of Mathematics Questionnaire (CMQ), the Mathematics Study Questionnaire (MSQ) and the Experience of Mathematics Questionnaire (EMQ) were administered to all students who attended the first mathematics tutorial in the first week of Semester 2. Again, students were given 30 minutes of class time to complete the questionnaires. There were approximately 850 returns.

5. The examination marks for the course were obtained from the School of Mathematics and Statistics.

The data set finally obtained and used in the subsequent analysis was the intersection of the above five data sets.

Results

A preliminary analysis has been carried out using the data obtained from the second semester questionnaires and the final mark in the subject. In these analyses, the surface strategy and surface intention subscales of the Mathematics Study Questionnaire (MSQ) were amalgamated, as were the deep intention and deep strategy subscales. Thus, the MSQ is comprised of two subscales; a Surface Approach subscale and a Deep Approach subscale.

Who were the Participants?

Of the 300 students in our final sample, 62% are male. One hundred and forty four students (48%) had completed 4 Unit mathematics for their final NSW secondary school examination, the Higher School Certificate (HSC). Almost all the others (44% of the students) had studied 3 Unit mathematics. The rest had studied mathematics at various levels in Australia or overseas. Hence, these 300 students would be considered successful in terms of their school education in mathematics, having almost all studied mathematics at the two highest levels of the five levels offered in NSW for matriculation.

Reliability of the Instruments

All subscales of the Conceptions of Mathematics Questionnaire (CMQ) and the Mathematics Study Questionnaire (MSQ) yielded α-coefficients greater than 0.77. For the Experience of Mathematics Questionnaire (EMQ), the Inappropriate Workload, Clear Goals and Good Teaching subscales were highly reliable ($\alpha > 0.74$) but the Independence subscale had a lower but acceptable reliability ($\alpha = 0.5$). A reliability analysis of the Inappropriate Assessment subscale indicated that it should be reduced from eight items to four items. The reduced subscale used in the data analysis provided a reliability of $\alpha = 0.72$.

Emerging Patterns

The correlations between the scores on the various subscales were calculated. The Pearson correlation matrix is shown in Table 2.

Table 2: Correlation Coefficients

	Frag. Concept	Co-hesive Concept	Surface Apprch	Deep Apprch	TER	Final Mark
CMQ Fragmented Conception	1					
CMQ Cohesive Conception	−0.11*	1				
MSQ Surface Approach	0.36**	−0.04	1			
MSQ Deep Approach	−0.12*	0.43**	−0.27**	1		
TER	−0.13*	0.18**	−0.25**	0.18**	1	
Final Mark in Math1	−0.17**	0.17**	−0.19**	0.23**	0.6**	1
EMQ Inappropriate Assessmt	0.19**	−0.12*	0.12*	−0.15**	−0.09	−0.06
EMQ Clear Goals	−0.03	0.24**	−0.17**	0.29**	0.12*	0.14*
EMQ Inappropriate Workload	0.18**	−0.12*	0.46**	−0.15**	−0.26**	−0.21**
EMQ Good Teaching	0.04	0.26**	−0.13*	0.41**	0.06	0.13*
EMQ Independence	−0.02	0.09	−0.17**	0.31**	−0.04	0.00

	Inappropriate Assessment	Clear Goals	Inappropriate Workload	Good Teaching
CMQ Fragmented Conception				
CMQ Cohesive Conception				
MSQ Surface Approach				
MSQ Deep Approach				
TER				
Final Mark in Math1				
EMQ Inappropriate Assessmt	1			
EMQ Clear Goals	−0.33**	1		
EMQ Inappropriate Workload	0.22**	−0.43**	1	
EMQ Good Teaching	−0.15*	0.41**	−0.21**	1
EMQ Independence	−0.00	0.22**	−0.08	0.41**

* indicates $p<0.05$, ** indicates $p<0.01$

Table 2 shows that the Fragmented Conception subscale was positively correlated to the Surface Approach subscale ($r=.36$, $p<0.01$). The Cohesive Conception subscale was positively correlated to the Deep Approach subscale ($r=.43$, $p<0.01$). Further, the Surface Approach subscale was also positively correlated with the Inappropriate Workload subscale ($r=.46$, $p<0.01$). The Deep Approach subscale, on the other hand, was also positively correlated with the Good Teaching and Independence subscales ($r\geq.31$, $p<0.01$).

There was a substantial positive correlation between the TER (the ranking used by universities to admit students) and the Final Mark in Mathematics 1 ($r=.6$, $p<.01$). Hence, rather than including both these variables in further analysis, only the Final Mark was used.

Other statistically significant correlations ($p<0.01$), though lower, were in the expected directions. For example, the Fragmented Conception subscale was correlated positively with the Inappropriate Assessment and the Inappropriate Workload subscales but negatively with the Final Mark. The

Cohesive Conception subscale was positively correlated with Final Mark and the Good Teaching and Clear Goals subscales.

A principal components factor analysis was carried out to explore the relationships between the variables. Analytic solutions were examined which extracted two factors, both with eigenvalues greater than one, as indicated by a scree test. This was followed by an oblique rotation. In accordance with common practice, the pattern matrix loadings were used to interpret the extracted factors (Entwistle & Ramsden, 1983). Table 3 gives the matrix of pattern loadings under an oblimin rotation.

Table 3: *Pattern Matrix Derived from Factor Analysis*

	Factor 1	Factor 2
CMQ Fragmented Conception		0.72
CMQ Cohesive Conception	0.52	
MSQ Surface Approach		0.73
MSQ Deep Approach	0.68	
EMQ Clear Goals	0.56	
EMQ Inappropriate Workload		0.68
EMQ Good Teaching	0.81	
EMQ Independence	0.65	
EMQ Inappropriate Assessment		0.44
Final Mark in Math1		–0.43

Loadings below 0.3 are omitted

The first factor shows that the Cohesive Conception, Deep Approach, Clear Goals, Good Teaching and Independence subscales were related to each other. Similarly, the second factor shows that the Fragmented Conception, Surface Approach, Inappropriate Workload and Inappropriate Assessment subscales were related to each other. The second factor loaded negatively (–0.43) with the Final Mark in Maths 1. Interestingly, the Final Mark did not load on Factor 1 on which the Cohesive Conception and Deep Approach subscales had high loadings.

Differences were found between the students who had studied mathematics at the 3 Unit level and those who had studied 4 Unit Mathematics for their HSC. Two-tailed t tests, showed (with $p<0.01$ in each case) that, on average, the 3 Unit students scored higher than the 4 Unit students on the Fragmented Conception, Surface Approach and Inappropriate Workload subscales and lower on the Independence and Deep Approach subscales and on the Final Mark in Mathematics 1.

Females and males differed on one subscale only — the Fragmented Conception subscale, on which the mean score for females was higher than the mean score for males ($t=2.8$, $p<0.01$).

Discussion

Summary of the Results

The results of this study support our earlier postulate. That is, the way students conceive of mathematics is related to their approaches to learning mathematics and to their perceptions of the teaching and learning environment.

There are two qualitatively different patterns that emerge from this study. The first suggests that conceiving of mathematics as a fragmented body of knowledge is related to surface approaches to learning mathematics, perceptions of assessment as measuring reproduction and perceptions that the workload is too high. Moreover, students with fragmented conceptions and surface approaches evidently do not achieve well in their mathematics studies at university.

On the other hand, cohesive conceptions of mathematics are associated with deep approaches to learning mathematics, perceptions of good teaching and clear subject goals and beliefs in independent learning. These students have very different interpretations of learning mathematics at university. Unfortunately, these students did not seem to be advantaged in their mathematical achievement. This may indicate that the traditional tests of achievement as used in this study are not a good measure of the understanding of mathematics.

Issues for University Mathematics Education

This study suggests that students' views of mathematical knowledge relate to their experiences of learning it as a whole. It emphasises the need to shift attention away from considering teaching and learning as independent activities to a more systemic view of the learning environment. That is, university teachers need to consider not only the content and the presentation of mathematics, but also how the students perceive it and their own learning.

Most students learn mathematics at school and university in a competitive environment with the emphasis on external assessment. This shapes their conceptions of mathematics and approaches to learning it. In addition, students who see these assessment requirements as measuring reproduction are also likely to believe that they have an onerous workload. Students may be diverted away from orientations that emphasise understanding and personal meaning toward an emphasis on satisfying what they perceive as the assessment requirements. Further, they will not do well in mathematics at university.

In our earlier study (Crawford et al, 1994a), we found that few students, when entering university, had a cohesive view of mathematics or adopted deep approaches to learning it. Unless university mathematical education addresses this, few students will recognise the possibilities for using mathematics in the different contexts in their lives. In an era of technology and mass information, when machines are increasingly used to carry out routine mathematical tasks in all fields, the challenge to higher education in mathematics is to develop the human potential for creativity and critical evaluation.

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